What do preservice teachers attend to when generalizing about an area strategy?

Valuing and leveraging student thinking is imperative for teaching mathematics equitably. Generalizations about nonstandard approaches can reveal the deep understanding of preservice teachers. Simulations can be an effective way to assess the validity of a nonstandard strategy for finding the area of a rectangle. How to account for non-unit sized parts in the rectangle (i.e., fractional parts).

Participants: 38 preservice elementary teachers at three stages in a two-year teacher education program: pre-admitted students, 50% of students in Year 1, and 67% of students in Year 2.

The simulation assessment consists of three parts:

- Preparation: Preparing for an interaction with a standardized student about a specific piece of student work.
- Simulation: Eliciting and probing the standardized student’s thinking to understand the steps the student took, why the student performed the steps, and the student’s understanding of the key mathematical ideas.
- Interview: Interpreting the student’s thinking and using evidence from the simulation. This includes generalizing about the validity of the student’s process:
  - What has to be true about a shape for this strategy to work?
  - What is a shape for which this strategy would not work? Why?

The generalization: Skillful mathematics teaching involves being able to size up children’s strategies and determine whether a strategy would work in general (Ball, Thames, & Phelps, 2008). This may be particularly challenging for novice teachers who have less familiarity with nonstandard approaches.

Decomposition of Practice: Teaching can be broken down into smaller parts that can be taught, studied, and rehearsed by preservice teachers. The parts must maintain their integrity so that they can be reintegrated into the practice of teaching (Grossman & Shahan, 2005).

Simulations: Situations that represent a context of practice with fidelity and elicit authentic professional work. Used in the preparation of professionals in other fields. Focus on the doing of teaching while standardizing important contextual factors that impact both teaching and ability to appraise its quality.

Most preservice teachers identified relevant problem features needed for the student’s process to work. 79% of preservice teachers clearly articulated the core idea that each row of the rectangle needs to contain the same number of unit squares.

The remaining 21% of preservice teachers discussed features that were not directly relevant to the core idea (e.g., having whole unit squares and no partial squares).

All participants accurately identified a shape for which the student’s strategy would not work.

Examples: Triangles, circles, trapezoids, composite shapes.

Explanations for why the student’s strategy would not work for a given shape varied.

54% of preservice teachers explained that the strategy would not work for a given shape by addressing the core idea — the shape lacks the same number of unit squares in each row.

Other explanations focused on the potential difficulty of visualizing unit squares or the presence of partial squares.

Attention to the properties of shapes and use of precise mathematical language varied widely (e.g., saying “bent edges” or “small corners” in reference to non-right or acute angles).

Participants who were further along in the teacher education program were more likely to articulate the core idea in their explanations: 40% of pre-admitted students, 50% of students in Year 1, and 67% of students in Year 2 articulated the core idea.

METHODS

- Participants: 38 preservice elementary teachers at three stages in a two-year teacher education program: pre-admission, beginning of Year 1, and beginning of Year 2.
- Simulation assessment.
- Analyzed preservice teachers’ generalizations focusing on:
  - the problem features needed for the approach to work;
  - the scenarios in which the approach would not work; and
  - explanations about why the approach would not work in such scenarios.

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CONCLUSIONS

Preservice teachers can use written work and interaction with a simulated student to generalize about problem features needed for a strategy to work and identify scenarios in which the strategy would not work.

Explaining when and why the strategy both will and will not work reveals different nuances in preservice teachers’ thinking and facilitates distinctions between cases of superficial and deep understanding.

Preservice teachers may experience challenges in generalizing about nonstandard approaches, such as:

- Identifying mathematical ideas at the core of the strategy.
- Using knowledge of properties of shapes to reason about how an area strategy might apply to a given problem.
- Using accurate and precise mathematical language to describe concepts of geometry and measurement.
- Further investigation is needed to explore the relationship between program learning experiences and differences in preservice teachers’ mathematical knowledge for teaching.